# Algebra 2

## 6-01 Exponent Properties and *e* (5.2, 6.2)

### Properties of Rational Exponents

- $x^m \cdot x^n = x^{m+n}$ (Product Property)
- $(xy)^m = x^m y^m$ (Power of a Product Property)
- $(x^m)^n = x^{mn}$ (Power of a Power Property)
- $\frac{x^m}{x^n} = x^{m-n}$ (Quotient Property)
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ (Power of a Quotient Property)
- $x^{-m} = \frac{1}{x^m}$ (Negative Exponent Property)

Simplify the expression. Write your answer using only positive exponents.

$\left(\frac{3w}{2x}\right)^4$ $6b^0$

### *e*

- Called the _____
- Found by putting really big numbers into $\left(1+\frac{1}{n}\right)^n$ = _____
- _____ number like π

### Simplifying natural base expressions

- Just treat *e* like a regular _____

$(5e^{7x})^4$ $\frac{11e^9}{22e^{10}}$

### Evaluate the natural base expressions using your calculator

Rewrite in the form $y = ab^x$

$y = e^{-0.75t}$ $y = 2e^{0.4t}$

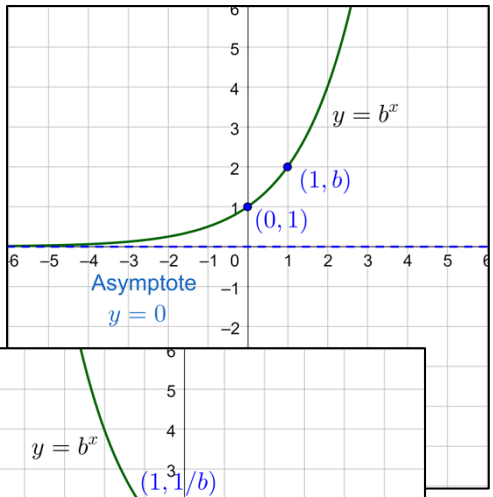
292#1-4; 305#1-9 odd; 11, 12; 25-28 all; 43, 45, 51, 53 (no graph), 55 (no graph) = 20

# Algebra 2

## 6-02 Exponential Growth and Decay Functions (6.1)

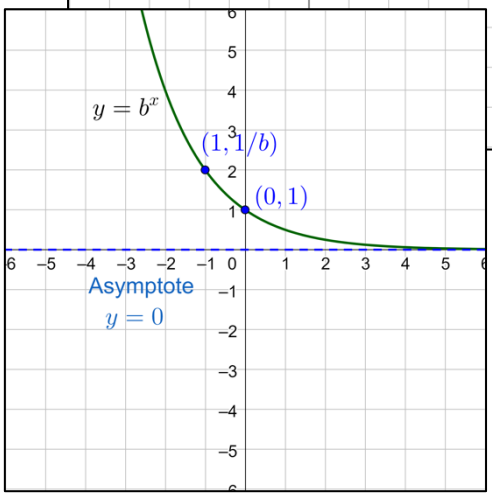
### Exponential Function

- $y = b^x$
- _____ ($b$) is a positive number other than _____




### Exponential Growth

- Always _____ and rate of change is _____
- _____
- $y$-intercept is _____
- Horizontal asymptote _____
- $b$ is the _____ factor

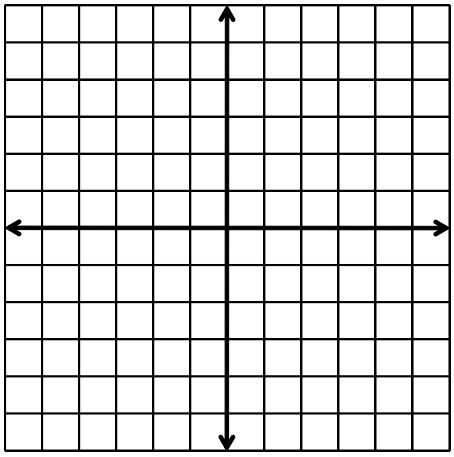



### Exponential Decay

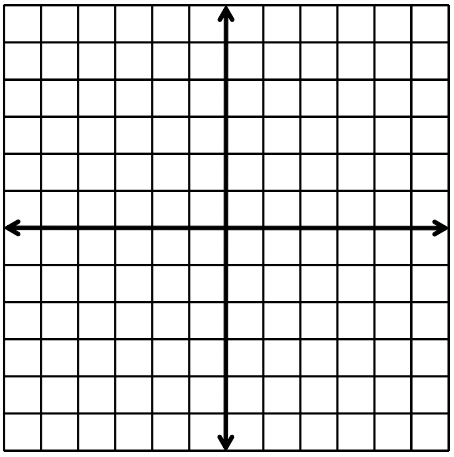
- Always _____ and rate of change is _____
- _____
- $y$-intercept is _____
- Horizontal asymptote _____
- $b$ is the _____ factor

Determine whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

$f(x) = \left(\frac{1}{6}\right)^x$



$y = \left(\frac{4}{3}\right)^x$



### Exponential Growth Model (word problems)

$$y = a(1 + r)^t$$

$y$ = current amount; $a$ = initial amount; $r$ = growth percent; $1 + r$ = growth factor; $t$ = time

## Exponential Decay Model (word problems)

$$y = a(1 - r)^t$$

$y$ = current amount; $a$ = initial amount; $r$ = decay percent; $1 - r$ = decay factor; $t$ = time

The population $P$ (in millions) of Peru during a recent decade can be approximated by $P = 28.22(1.01)^t$, where $t$ is the number of years since the beginning of the decade.

(a) Determine whether the model represents exponential growth or decay

(b) Identify the annual percent increase or decrease in population

(c) Estimate when the population was about 30 million

The value of a mountain bike $y$ (in dollars) can be approximated by the model $y = 200(0.65)^t$, where $t$ is the number of years since the bike was purchased.

(a) Determine whether the model represents exponential growth or decay

(b) Identify the annual percent increase or decrease

(c) Estimate when the value of the bike will be $50

## Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A$ = amount at time $t$; $P$ = principle (initial amount); $r$ = annual rate; $n$ = number of times interest is compounded per year

Find the balance in the account earning compound interest after 6 years when the principle is $3500.

$r$ = 2.16%, compounded quarterly

298#7-15 odd, 19-22, 44, 35, 39, 40, 41, 42, 53, 54, 55, 61, 63 = 20

# Algebra 2

## 6-03 Rewrite Exponential as Logarithmic Functions (6.3)

### Logarithms

- Logarithms are _____
- $\log_b a =$ _____ of $b$ to get $a$

$\log_3 81$

$\log_3 3$

### Calculator has two logs

- _____ Log: log = $\log_{10}$
- _____ Log: ln = $\log_e$
- (Some calculators can do log of any base.)

$\log 6$

$\ln \frac{1}{3}$

### Definition of Logarithm with Base *b*

$$\log_b y = x \Leftrightarrow b^x = y$$

- Read as "log base $b$ of $y$ equals $x$"
- Logs = _____!!
- Logs and exponentials are _____
- They _____ each other
- They _____ each other out

Rewrite as an exponential: $\log_3 9 = 2$

Rewrite as a log: $6^2 = 36$

### Simplify log expressions

If exponential with base $b$ and log with base $b$ are inside each other, they _____

$7^{\log_7 x}$

$\log_3 3^{2x}$

312#1, 3, 5, 7, 9, 11, 13, 15, 17, 23, 25, 31, 33, 35, 37, 75, 77, 79, 83, 85 = 20

# Algebra 2

## 6-04 Logarithmic Properties (6.5)

### Properties of Logarithms

- Product Property

$$\log_b uv = \log_b u + \log_b v$$

- Quotient Property

$$\log_b \frac{u}{v} = \log_b u - \log_b v$$

- Power Property

$$\log_b u^n = n \log_b u$$

### Expand logarithms

- Rewrite as _____ logs

$\log 10x^5$

$\ln \frac{x}{3y}$

### Condense logs

- Try to write as a _____ log

$\log_5 4 + \frac{1}{3}\log_5 x$

$6 \ln x + 4 \ln y$

### Change-of-Base Formula

$$\log_c u = \frac{\log_b u}{\log_b c}$$

- This lets you evaluate any log on a _____

Evaluate $\log_9 15$

Evaluate $\log_4 7$

327#11-17 odd; 21-27 odd; 29-35 odd; 37-38; 46, 47, 51, 57, 59, 61 = 20

# Algebra 2

## 6-05 Graph Exponential and Logarithmic Functions (6.4)

### Exponential Function

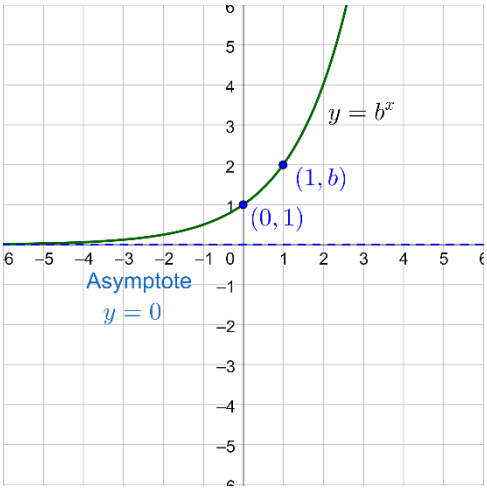
$$y = b^x$$

- _____ ($b$) is a positive number other than 1

In general

$$y = ab^{cx-h} + k$$

- $a$ is _____ stretch
  - If $a$ is –, _____ over $x$-axis
- $c$ is _____ shrink by $\frac{1}{c}$
  - If $c$ is –, _____ over $y$-axis
- $h$ is _____ shift
- $k$ is _____ shift
- Horizontal asymptote: _____

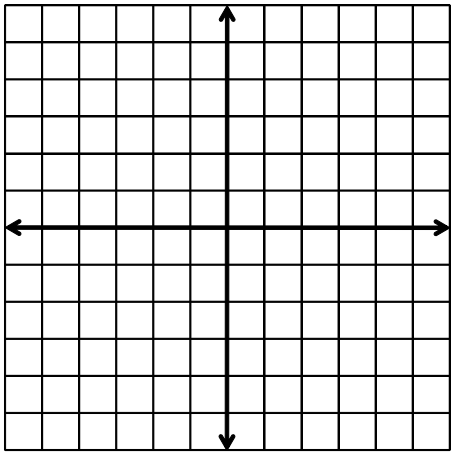



### Graph Exponential Functions

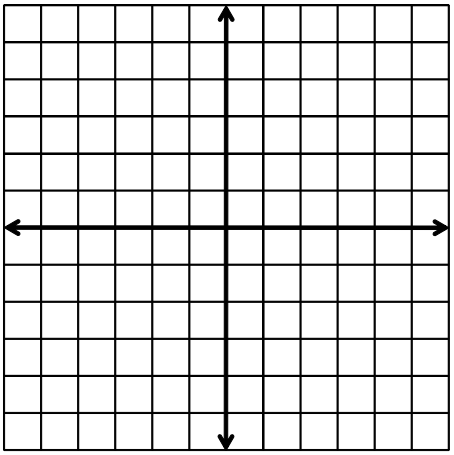
1. Find and graph the _____
2. Make a _____
3. _____ points and draw the curve
4. Make sure the curve is near the _____ at the _____ of the graph

(a) Describe the transformations. (b) Then graph the function.

$g(x) = -2^{x-3}$



$g(x) = e^{2x}$



### Logarithmic Function

$$y = \log_b x$$

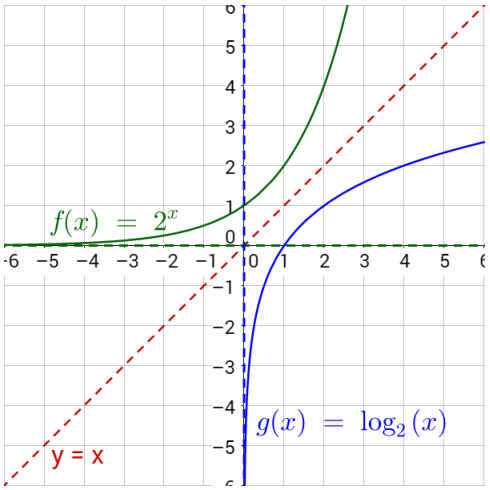
- _____ ($b$) is a positive number other than 1
- Logarithms and exponentials are _____

Algebra 2 6-05 Name: _____

In general

$$y = a\log_b(cx - h) + k$$

- $a$ is _____ stretch
  - If $a$ is –, _____ over $x$-axis
- $c$ is _____ shrink by $\frac{1}{c}$
  - If $c$ is –, _____ over $y$-axis
- $h$ is _____ shift
- $k$ is _____ shift
- Vertical asymptote: _____

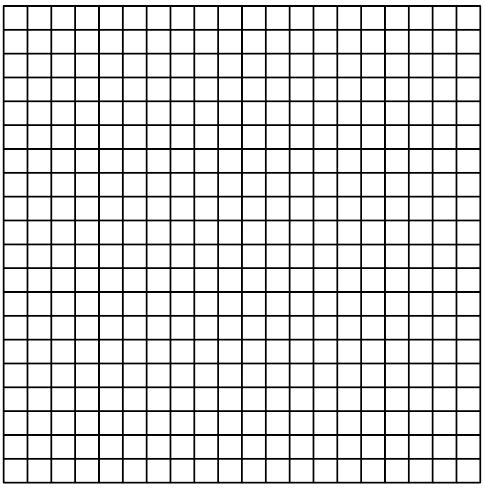



## Graph Logarithmic Functions

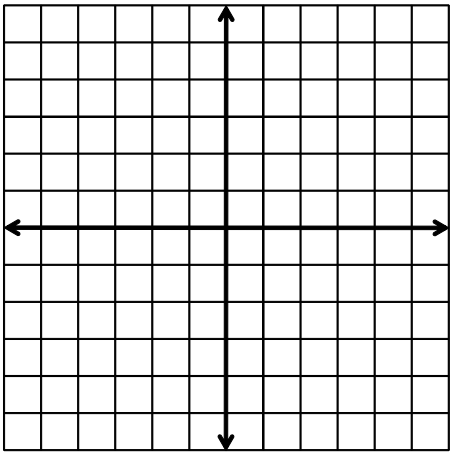
1. Find and graph the _____
2. Make a _____
   a. You may need to use the _____ formula
3. _____ points and draw the curve
4. Make sure the curve is near the _____ at the _____ of the graph

(a) Describe the transformations. (b) Then graph the function.

$g(x) = -\log_{1/5}(x - 7)$



$g(x) = 3\log_4 x - 5$



## Find the inverse

1. _____ log or exponential part
2. _____ $x$ and $y$
3. Then _____ as exponential or log

$y = \ln(x - 1)$

$y = 5^x - 9$

320#15, 17, 21; 313#57, 59; 320#25, 27; 313#43, 45, 47, 51; 322# 53, 55, 62, 65 = 15

# Algebra 2

6-06 Solve Exponential and Logarithmic Equations (6.6)

## Solving Exponential Equations

**Method 1) if the _____ are equal, then _____ are equal**

$5^{x-3} = 25^{x-5}$

$2^{3x+5} = 2^{1-x}$

**Method 2) take _____ of both sides**

$5(7)^{5x} = 60$

$3e^{4x} + 9 = 15$

Algebra 2 6-06 Name: _____

**Solving Logarithmic Equations**

**Method 1) if the _____ are equal, then _____ are equal**

$\ln(4x - 12) = \ln x$

$\log_2(3x - 4) = \log_2 5$

**Method 2) _____ both sides**

- Make both sides exponents with the base of the log

$\log_2(4x + 8) = 5$

$\log_3(2x + 1) = 2$

334#1, 3, 5, 7, 9, 11, 13, 17, 19, 21, 22, 23, 25, 27, 29, 75, 77, 79, 83, 87 = 20

# Algebra 2

## 6-07 Modeling with Exponential and Logarithmic Functions (6.7)

### Choosing Functions to Model Data

- For _____ spaced *x*-values
  - If *y*-values have common ratio (multiple) → _____
  - If *y*-values have finite differences → _____

Determine the type of function represented by each table.

| *x* | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| *y* | 4 | 3 | 7 | 16 | 30 | 49 |

| *x* | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| *y* | 0.25 | 1 | 4 | 16 | 64 | 256 |

### Use the regression feature on a graphing calculator

#### TI-84

1. Enter points in STAT → EDIT
2. To see points go Y= and highlight Plot1 and press ENTER to keep it highlighted
3. Press Zoom and choose ZoomStat
4. Go to STAT → CALC → ExpReg for exponential OR LnReg for logarithmic

#### NumWorks

1. Choose Regression from homescreen
2. In Data tab, enter points
3. Go to Graph tab
4. To change regression type, press OK and choose a different regression
5. Read the answer off the bottom of the graph

Determine whether the data show an exponential relationship. Then write a function that models the data.

| *x* | −3 | −1 | 1 | 3 | 5 |
|---|---|---|---|---|---|
| *y* | 2 | 7 | 24 | 68 | 194 |

| *x* | 1 | 6 | 11 | 16 | 21 |
|---|---|---|---|---|---|
| *y* | 12 | 28 | 76 | 190 | 450 |

342#1-4, 19, 20, 21, 22, 30, 31, 32, 39, 41, 47, 49 = 15

# Algebra 2

## 6-Review

**Take this test as you would take a test in class. When you are finished, check your work against the answers.**

**6-01**

**Simplify**

1. $\frac{e^2}{2e^3e^{-2}}$
2. $3e^2 - 7e^2$
3. $2\left(\frac{3}{4}\right)^3$

**6-02**

4. Determine whether $f(x) = 2\left(\frac{1}{2}\right)^{x-1} + 4$ is exponential growth or exponential decay.

**6-02**

**Solve the word problems. Round to two decimal places.**

5. You charge $1200 on a credit card that charges 20% interest compound daily. If you do not make a payment, how much will you owe after 1 year?
6. A rabbit population starts with 20 individuals. If the population increases 30% every year, estimate the number of rabbits in the population after 5 years.

**6-03**

7. Rewrite $10^2 = 100$ as a logarithm.

**Evaluate.**

8. $\log_4 256$
9. $\log_2 \frac{1}{1024}$

**6-04**

**Condense the expression.**

10. $\ln 12 + 3\ln x - \ln x^2$

**Expand the logarithm.**

11. $\ln \frac{2x^7}{y^2}$

**Use the change-of-base formula to evaluate the logarithm. (Round to three decimal places.)**

12. $\log_4 150$
13. $\log_{17} 1321$

**6-05**

**Describe the transformations from *f*(*x*) to *g*(*x*).**

14. $f(x) = 2^x; g(x) = -2^x - 1$
15. $f(x) = \ln x\,; g(x) = 2\ln(-x) - 3$
16. Write a function that is the transformation of $f(x) = \log_2 x$ with a vertical stretch by a factor of 3 and a translation 4 left.

**6-05**

**Graph and state the domain and range.**

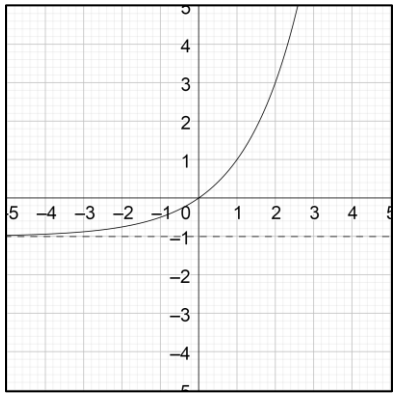
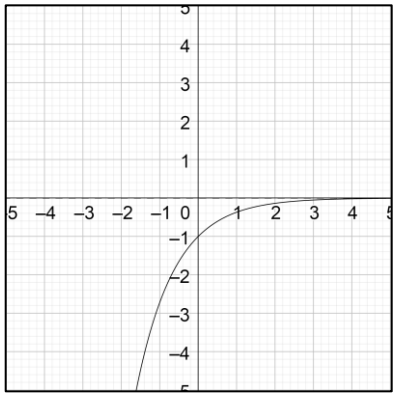
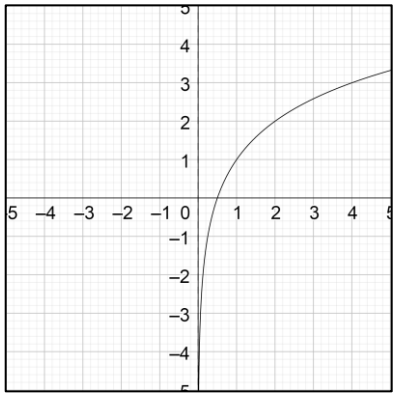
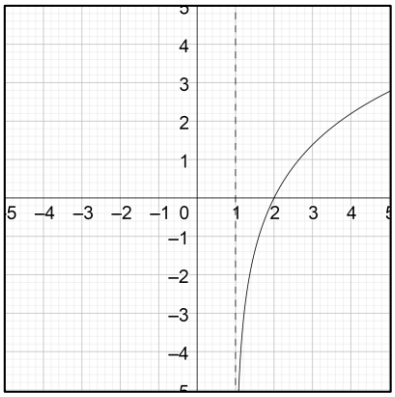
17. $y = 2^x - 1$
18. $y = -e^{-x}$
19. $y = \log_2 x + 1$
20. $y = 2\ln(x - 1)$

**6-06**

**Solve. (Round to three decimal places.)**

21. $4^{2x+1} = 32^{x-1}$
22. $7^{x+4} + 3 = 51$
23. $\log_{21}(2x + 17) = \log_{21}(x - 1)$
24. $\log_5(2x + 7) = 15$

## Answers

1. $\frac{e}{2}$
2. $-4e^2$
3. $\frac{27}{32}$
4. Exponential decay
5. \$1465.60
6. 74 rabbits
7. $\log_{10} 100 = 2$
8. 4
9. $-10$
10. $\ln(12x)$
11. $\ln 2 + 7\ln x - 2\ln y$
12. 3.614
13. 2.536
14. Reflection over the x-axis, translation 1 down
15. Vertical stretch by factor of 2, reflection over y-axis, translation 3 down
16. $g(x) = 3\log_2(x+4)$
17. 
18. 
19. 
20. 
21. 7
22. $-2.011$
23. No solution ($-18$ is extraneous)
24. $1.526 \times 10^{10}$